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Research Paper

Thermoelastic Vibrations of Nonlocal Nanobeams Resting on a Pasternak Foundation via DPL Model

M.E. Nasr[1,2], Ahmed E. Abouelregal[1,3], A. Soleiman[1,2], K.M. Khalil[1,2]

[1] Department of Mathematics, College of Science and Arts, Jouf University, Gurayat, Saudi Arabia

[2] Department of Mathematics, Faculty of Science, Benha University, Benha 13518, Egypt, Email(s): mohamed.naser@fsc.bu.edu.eg, amrsoleiman@yahoo.com, halil.khalil@fsc.bu.edu.eg

[3] Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt, Email: ahabogal@gmail.com

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Corresponding author: M.E. Nasr (mohamed.naser@fsc.bu.edu.eg)

Abstract. The present work introduces the thermoelastic vibrations of nonlocal nanobeams resting on a two-parameter foundation. The governing equations are formulated for linear Winkler–Pasternak foundation type based on the generalized dual-phase-lag heat conduction and nonlocal beams theories. The nanobeam is subjected to a temperature ramping function. The coupled equations of the problem are formulated and solved by Laplace transform technique. The effects of the nonlocal parameter and different foundation parameters on the field variables are illustrated graphically and discussed. The results obtained are consistent with previous analytical and numerical results.

Keywords: Nanobeams, Nonlocal thermoelasticity, Ramp-type heating, Foundation.

1. Introduction

Nanotechnology is concerned with the manufacturing of advanced materials in the nanoscale, which offers a new class of structures with innovative properties and improved performance devices. Among these nanostructures, some nanobeams attract a greater concentration due to their various possible applications, such as nanowires, nano-probes, atomic power supply, nano-actuators, and nano-sensors. Also, nanoscale effects are significantly on the mechanical performance of nanostructures in where the sizes are small and similar to molecular distances. This inspired many researchers to find a new model to predict the mechanical behavior of these nanostructures.

Recently, there has been an increasing number of studies on nonlocal theoretical models, which include various types of nonlocal elasticity approaches consisting of hardening and softening models that are extensively investigated. The theory of nonlocal elasticity (NET) is applied for modeling of micro/nano-scale mechanical systems due to their generalization and simplicity. This theory was first introduced by Eringen [1-3]. The NET theory states that the stress field at any point in the body is a function of the strain field at each point of the continuum object. Many studies have used the nonlocal elasticity theory (NET) to take into account the vibration and nanoscale effects on the nanobeams and nanostructures. Some of them are found in references [4–23].

Nanobeams resting on elastic foundations are usually included in the design of aircraft structures and have wide applications in structural analysis. This inspired many scientists to examine the performance of structures in different kinds of elastic foundations. The Winkler-type elastic foundation is estimated as a series of closely spaced, mutually independent, and vertical linear elastic springs. The Pasternak model is a two-parameter model consists of a Winkler-type elastic spring and transverse shear deformation. The effect of Winkler and Pasternak elastic foundations on bending and vibration of micro-nano materials has been investigated by several authors [24-30].

The above investigations clearly show that most of the studies presented in the literature are related to the nonlocal and elastic foundation, but studies on the nonlocal and thermoelastic vibration are very limited. Only a few articles are available in the literature relating to thermoelastic foundation-supported nanobeams, where the generalized thermoelasticity theories are employed for the mathematical formulation of the problem. In the current paper, we investigate the thermoelastic vibration of a nanobeam resting on the Winkler-Pasternak foundation using Eringen's nonlocal elasticity and the thermoelastic dual-phase-lag model proposed by Tzou [31-33] in which the Fourier law is modified. The nanobeam is thermally loaded by ramp-type varying heat. The thermoelastic vibration of the temperature, deflection, the displacement and the bending moment of the nanobeam subjected to ramp-type varying heat are investigated. Some comparisons were presented graphically to estimate the effects of the nonlocal parameter and Winkler-Pasternak foundation parameters in all the field variables. The results in this work are intended to be useful for design, electromechanical applications and many areas of the industrial revolution.

2. Nonlocal Thermoelasticity with Phase Lags

According to the nonlocal elasticity theory of Eringen, the nonlocal differential constitutive equations for homogenous thermoelastic materials is [1-3]

$$(1-\xi\nabla^2)\sigma_{ij}=\tau_{ij} \tag{1}$$

where σ_{ij} and τ_{ij} are the nonlocal and local stress tensors, respectively. One may see that when the internal characteristic length is neglected, i.e., the particles of a medium are considered to be continuously distributed, ξ is zero, and eq. (1) reduces to the constitutive equation of classical local thermoelasticity.

The generalized heat equation with phase-lags proposed by Tzou [31-33] is given by

$$K\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\nabla^2\theta=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2}\right)\left[\rho C_E\frac{\partial\theta}{\partial t}+\gamma T_0\frac{\partial}{\partial t}(\mathrm{div}(u))-Q\right] \tag{2}$$

Constitutive equations:

$$\tau_{ij}=2\mu e_{ij}+\lambda e_{ij}-\gamma\theta\delta_{ij} \tag{3}$$

Equation of motion:

$$\sigma_{ji,j}+F_i=\rho\ddot{u}_i \tag{4}$$

Equations (1) and (2) describe the nonlocal thermoelectricity theory. It can be seen that the corresponding local thermoelasticity is recovered by putting $\xi=0$ in eq. (1).

3. Formulation of the Problem

Consider a thermoelastic nanobeam with $0\le x\le L$, $0\le z\le h$, where L and h are the length and thickness of the nanobeam. This nanobeam initially at temperature T_0 and resting on a linear Winkler-Pasternak foundation K_w and K_s as illustrated in Fig. 1. Also, we consider the x axis is drawn along the axial direction of the beam and y, z axes correspond to the width and thickness, respectively.

The displacement components are given by

$$u=-z\frac{\partial w}{\partial x},\ \nu=0, w(x,y,z,t)=w(x,t) \tag{5}$$

For a one-dimensional problem, the differential form of the constitutive eq. (3) after using eqs. (1) and (5) can be expressed as [8, 33]:

$$\sigma_x-\xi\frac{\partial^2\sigma_x}{\partial x^2}=-E\left[z\frac{\partial^2 w}{\partial x^2}+\alpha_T\theta\right] \tag{6}$$

where σ_x is the nonlocal axial stress, and $\alpha_T=\alpha_t/(1-2\nu)$.

As is known, the Winkler model of elastic foundation is the most preliminary in which the vertical displacement is assumed to be proportional to the contact pressure at an arbitrary point [35]. Due to the interaction between the nanobeam and the supporting foundation, the normal stress per unit area R_f (foundation reaction) and vertical displacement w at an arbitrary point on the lower boundary of the nanobeam hold the following relation [36, 37]

$$R_f=K_w w(x,t)-K_s\frac{\partial^2 w(x,t)}{\partial x^2} \tag{7}$$

where K_w is the Winkler's foundation constant, which is known as the modulus of the subgrade reaction, and K_s is the shear foundation modulus. It is noted that when $K_s=0$, eq.(8) is equivalent to that of the nanobeam on a Winkler foundation type; also, when $K_w=K_s=0$ (the subgrade reactions are zero), indicating that the nanobeam beam does not have a foundation. The equation of motion for transverse vibration of nanobeams can be written as

$$\frac{\partial^2 M}{\partial x^2}-R_f=\rho A\frac{\partial^2 w}{\partial t^2} \tag{8}$$

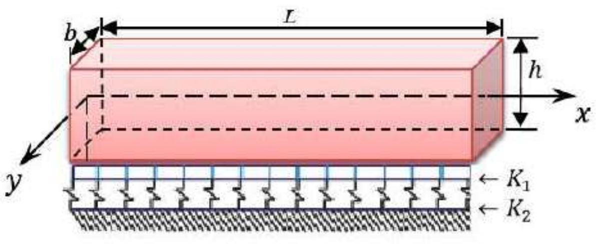

Fig. 1. Geometry of the nanobeam on Pasternak foundation.

With aid of eq. (6), the flexure moment is given by

$$M(x,t)-\xi\frac{\partial^2 M}{\partial x^2}=-IE\left[\frac{\partial^2 w}{\partial x^2}+\alpha_T M_T\right] \tag{9}$$

where

$$M_T=\frac{12}{h^3}\int_{-h/2}^{h/2}\theta(x,z,t)\,z\,dz \tag{10}$$

Also, it is exactly seen that the flexure moment of the nonlocal nanobeams is given by

$$M(x,t)=\xi A\rho\frac{\partial^2 w}{\partial t^2}+\xi K_w w(x,t)-(IE+\xi K_s)\frac{\partial^2 w(x,t)}{\partial x^2}-\alpha_t M_T \tag{11}$$

Substituting eq. (11) into eq. (8), one can get the motion equation of the nanobeam as

$$\frac{\partial^4 w}{\partial x^4}-\beta_1\frac{\partial^2 w}{\partial x^2}+\beta_2\frac{\partial^2}{\partial t^2}\left(w-\xi\frac{\partial^2 w}{\partial x^2}\right)+\beta_3 w+\beta_4\frac{\partial^2 M_T}{\partial x^2}=0, \tag{12}$$

where

$$\beta_1=\frac{\xi K_w+K_s}{IE+\xi K_s},\quad \beta_2=\frac{\rho A}{IE+\xi K_s},\quad \beta_3=\frac{K_w}{IE+\xi K_s},\quad \beta_4=\frac{\alpha_T}{IE+\xi K_s}. \tag{13}$$

The heat conduction eq. (2) can be written as follows ($Q=0$):

$$\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial z^2}\right)=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2}\right)\left[\frac{\rho C_E}{K}\frac{\partial\theta}{\partial t}-\frac{\gamma T_0}{K}z\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}\right)\right] \tag{14}$$

4. Solution of the Problem

We consider the nanobeam is thermally insulated, so that $\partial\theta/\partial z$ should vanish at the upper and lower surfaces of the nanobeam $z=\pm h/2$. Also, we assume that the increment temperature varies in a sinusoidal form along the thickness direction as

$$\theta(x,z,t)=\Theta(x,t)\sin\left(\frac{\pi z}{h}\right) \tag{15}$$

Substituting eq. (15) into eq. (12), one can get the motion equation of the nanobeams as

$$\frac{\partial^4 w}{\partial x^4}-\beta_1\frac{\partial^2 w}{\partial x^2}+\beta_2\frac{\partial^2}{\partial t^2}\left(w-\xi\frac{\partial^2 w}{\partial x^2}\right)+\beta_3 w+\frac{24\beta_4}{h\pi^2}\frac{\partial^2\Theta}{\partial x^2}=0 \tag{16}$$

Also, the flexure moment can be determined from eqs. (11) and (15) as

$$M(x,t)=\xi A\rho\frac{\partial^2 w(x,t)}{\partial t^2}+\xi K_w w(x,t)-(IE+\xi K_s)\frac{\partial^2 w(x,t)}{\partial x^2}-\frac{24\alpha_t}{h\pi^2}\Theta \tag{17}$$

From eqs. (14) and (15), the generalized heat conduction equation become

$$\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\Theta}{\partial x^2}-\frac{\pi^2}{h^2}\Theta\right)=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2}\right)\left[\frac{\rho C_E}{K}\frac{\partial\Theta}{\partial t}-\frac{\gamma T_0\pi^2 h}{24K}\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}\right)\right] \tag{18}$$

To facilitate the numerical analysis, dimensionless parameters are introduced. Now, for simplicity we will use the following non-dimensional variables:

$$\begin{gathered}\{x',z',u',w',h'\}=\frac{1}{L}\{x,z,u,w,h\},\ \{t',\tau_0',\tau_\theta',\tau_q'\}=\frac{c_0}{L}\{t,\tau_0,\tau_\theta,\tau_q\},\\ \xi'=\frac{\xi}{L^2},\quad \Theta'=\frac{\Theta}{T_0},\quad c_0 L=\frac{K}{\rho C_E},\quad M'=\frac{M}{ALE},\quad c_0=\sqrt{\frac{E}{\rho}}.\end{gathered} \tag{19}$$

So, the basic eqs. (16), (17) and (18) in nondimensional forms are simplified as (dropping the primes for convenience)

$$\frac{\partial^4 w}{\partial x^4}-A_1\frac{\partial^2 w}{\partial x^2}+A_2\left(\frac{\partial^2 w}{\partial t^2}-\xi\frac{\partial^4 w}{\partial t^2\partial x^2}\right)+A_3 w=-A_4\frac{\partial^2\Theta}{\partial x^2} \tag{20}$$

$$M(x,t)=\xi\frac{\partial^2 w(x,t)}{\partial t^2}+A_5 w(x,t)-A_6\frac{\partial^2 w(x,t)}{\partial x^2}-A_7\Theta \tag{21}$$

$$\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\Theta}{\partial x^2}-\frac{\pi^2}{h^2}\Theta\right)=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2}\right)\left[\frac{\partial\Theta}{\partial t}-\frac{\gamma T_0\pi^2 h c_0 L}{24K}\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}\right)\right] \tag{22}$$

where

$$A_1 = L^2\beta_1, \quad A_2 = L^2\beta_2 c_0^2, \quad A_3 = L^4\beta_3, \quad A_4 = \frac{24\beta_4 T_0}{h\pi^2}, \qquad A_5 = \frac{\xi L^2 K_w}{AE}, \quad A_6 = \frac{IE + \xi L^2 K_s}{AEL^2}, \quad A_7 = \frac{24 T_0 \alpha_t}{AEL^2 h\pi^2}. \tag{23}$$

5. Initial and Boundary Conditions

To solve the problem, the initial and boundary conditions must be taken into consideration. The homogeneous initial conditions are taken as

$$\Theta(x,0) = \frac{\partial \Theta(x,0)}{\partial t} = 0 = w(x,0) = \frac{\partial w(x,0)}{\partial t} \tag{24}$$

We will assume that the two ends of the nanobeam satisfy

$$w(0,t) = w(L,t) = 0 = \frac{\partial^2 w(0,t)}{\partial x^2} = \frac{\partial^2 w(L,t)}{\partial x^2} \tag{25}$$

Also, we consider the first end nanobeam is loaded thermally by ramp-type heating as

$$\Theta(x,t) = \Theta_0 \begin{cases} 0, & t<0, \\ \dfrac{t}{t_0}, & 0 \le t \le t_0, \\ 1, & t > t_0, \end{cases} \tag{26}$$

where t_0 is a non-negative constant called ramp-type parameter and Θ_0 is a constant. In addition, the temperature at the end boundary should satisfy the following relation

$$\frac{\partial \Theta}{\partial x} = 0 \quad \text{on} \quad x = L \tag{27}$$

6. Solution in the Laplace Transform Domain

The closed form solution of the governing and constitutive equations can be possible by adapting the Laplace transform method. Taking the Laplace transform defined by the relation,

$$\bar{f}(x,s) = \int_0^\infty f(x,t)e^{-st}dt \tag{28}$$

to both sides of eqs. (20), (21) and (22) and using the homogeneous initial conditions (24), one gets the field equations in the Laplace transform space as

$$\frac{d^4\bar{w}}{dx^4} - A_{10}\frac{d^2\bar{w}}{dx^2} + A_{11}\bar{w} = -A_4\frac{d^2\bar{\Theta}}{dx^2}, \tag{29}$$

$$M(x,t) = A_{12}\bar{w} - A_6\frac{d^2\bar{w}}{dx^2} - A_7\bar{\Theta}, \tag{30}$$

$$\left(\frac{d^2}{dx^2} - B_1\right)\bar{\Theta} = -B_2\frac{d^2\bar{w}}{dx^2}, \tag{31}$$

where

$$A_{10} = (A_1 + s^2\xi A_2), \quad A_{11} = (A_3 + s^2 A_2), \quad A_{12} = (\xi s^2 + A_7), \quad A_{13} = \frac{\pi^2}{h^2}, \qquad A_{14} = \frac{\gamma T_0 \pi^2 h c_0 L}{24K}, \quad B_1 = A_{13} + \frac{s\left(1+\tau_q s + \frac{1}{2!}\tau_q^2 s^2\right)}{1+\tau_\theta s}, \quad B_2 = \frac{sA_{14}\left(1+\tau_q s + \frac{1}{2!}\tau_q^2 s^2\right)}{1+\tau_\theta s}. \tag{32}$$

Elimination $\bar{\Theta}$ or $\bar{w}$ from eqs. (29) and (31), one obtains:

$$(D^6 - AD^4 + BD^2 - C)\{\bar{\Theta}, \bar{w}\}(x) = 0, \tag{33}$$

where the coefficients A, B and C are given by

$$A = A_4 B_2 + A_{10} + B_1, \quad B = A_{11} + B_1 A_{10}, \quad C = B_1 A_{11}, \quad D = \frac{d}{dx} \tag{34}$$

Equation (33) can be moderated to

$$(D^2-m_1^2)(D^2-m_2^2)(D^2-m_3^2)\{\bar{\Theta},\bar{w}\}(x)=0, \tag{35}$$

where $m_n^2, n=1,2,3$ are roots of

$$m^6-Am^4+Bm^2-C=0, \tag{36}$$

The solution of the governing eq. (36) in the Laplace transformation domain can be represented as

$$\{\bar{\Theta},\bar{w}\}(x)=\sum_{n=1}^{3}\left(\{1,F_n\}C_n e^{-m_n x}+\{1,F_{n+3}\}C_{n+3}e^{m_n x}\right) \tag{37}$$

Where the compatibility between these two equations and eq. (31), gives

$$F_n=\Gamma_n C_n,\quad \Gamma_n=-\frac{B_2\, m_n^2}{m_n^2-B_1} \tag{38}$$

where C_n and F_n are some parameters depending on s. From eqs. (15) and (31), we get

$$\bar{\theta}(x)=\sin\left(\frac{\pi z}{h}\right)\sum_{n=1}^{3}\left(F_n C_n e^{-m_n x}+F_{n+3}C_{n+3}e^{m_n x}\right) \tag{39}$$

The axial displacement after using eq. (37) takes the form

$$\bar{u}(x)=-z\frac{d\bar{w}}{dx}=z\sum_{n=1}^{3}m_n\left(C_n e^{-m_n x}-C_{n+3}e^{m_n x}\right) \tag{40}$$

Substituting the expressions of $\bar{w}$ and $\bar{\Theta}$ from eq. (37) into eq. (30), we get at the solution for the bending moment $\bar{M}$ as follows

$$\bar{M}(x)=\sum_{n=1}^{3}\left(A_{12}-A_6 m_n^2-A_7\Gamma_n\right)\left(C_n e^{-m_n x}+C_{n+3}e^{m_n x}\right) \tag{41}$$

In addition, the strain will be

$$\bar{e}(x)=\frac{d\bar{u}}{dx}=-z\sum_{n=1}^{3}m_n^2\left(C_n e^{-m_n x}+C_{n+3}e^{m_n x}\right) \tag{42}$$

After using Laplace transform, the boundary conditions (25) - (27) take the forms

$$\begin{aligned}
&\bar{w}(0,s)=\bar{w}(L,s)=0,\\
&\frac{\partial^2\bar{w}(0,s)}{\partial x^2}=\frac{\partial^2\bar{w}(L,s)}{\partial x^2}=0,\\
&\bar{\Theta}(0,s)=\Theta_0\left(\frac{1-e^{-st_0}}{s^2 t_0}\right)=\bar{G}(s),\\
&\frac{d\bar{\Theta}(L,s)}{dx}=0.
\end{aligned} \tag{43}$$

Substituting eq. (37) into the above boundary conditions, one obtains six linear equations

$$\sum_{n=1}^{3}(C_n+C_{n+3})=0, \tag{44}$$

$$\sum_{n=1}^{3}\left(C_n e^{-m_n L}+C_{n+3}e^{m_n L}\right)=0, \tag{45}$$

$$\sum_{n=1}^{3}m_n^2(C_n+C_{n+3})=0, \tag{46}$$

$$\sum_{n=1}^{3}m_n^2\left(C_n e^{-m_n L}+C_{n+3}e^{m_n L}\right)=0, \tag{47}$$

$$\sum_{n=1}^{3}(\Gamma_n C_n+\Gamma_{n+3}C_{n+3})=\bar{G}(s), \tag{48}$$

$$\sum_{n=1}^{3}m_n\left(\Gamma_n C_n e^{-m_n L}-\Gamma_{n+3}C_{n+3}e^{m_n L}\right)=0, \tag{49}$$

The solution of the above system of linear equations gives the unknown parameters C_n, $(n=1,2,...,6)$. To determine the studied fields in the physical domain, the Riemann-sum approximation method is used to obtain numerical results. The details of these methods can be found in [32].

7. Special Cases

The following special cases can be obtained from the system of eqs. (11), (12) and (14):

- The equations of a coupled nonlocal thermoelasticity theory result from eq. (14), in the limiting case $\tau_q = \tau_\theta = 0$.
- The equations of coupled local models of thermoelasticity result from eqs. (11), (12) and (14) in the limiting case by letting $\xi = 0$.
- The equations of nonlocal nanobeams resting on Winkler foundation type result from eqs. (11), (12) and (14) in the limiting case by letting $K_s = 0$.
- When $K_w = K_s = 0$ (the subgrade reactions are zero), indicating that the nanobeam does not have a foundation.

8. Numerical Results

To check the validity of the obtained formulations, the predicted results are compared with those available in the open literature. The influences of various parameters such as the material length scale, ramping time parameters and the foundation stiffness and foundation shear elastic on the flexure moment M, temperature θ, displacement u and lateral vibration w are investigated. The material properties of the silicon are taken in the numerical simulations [38].

$$K = 156\ \mathrm{Wm^{-1}K^{-1}},\quad E = 169\mathrm{GPa},\quad \rho\text{=}2330\ \mathrm{kgm^{-3}},\quad \nu = 0.22,\quad \alpha_t = 2.59\times10^{-6}\,\mathrm{K^{-1}},\quad C_E = 713\,\mathrm{J/kgK},\quad T_0 = 293\mathrm{K}.$$

The aspect ratios of the nanobeam are fixed as $L/h = 10$ and $b/h = 0.5$. The numerical results are presented graphically in Figs. 2-9 at different positions x in the wide range of $0.0 \le x \le 1.0$ when $t = 0.12, L = 1$ and $z = h/3$. Numerical calculations and graphs have been divided into two cases.

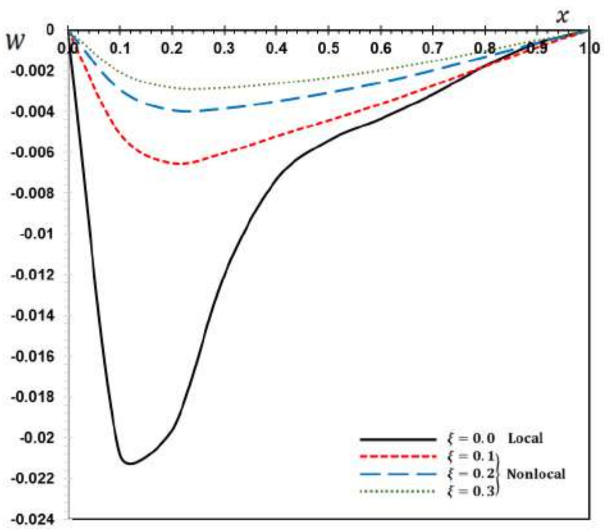

Fig. 2. The transverse deflection w with different nonlocal parameter ξ

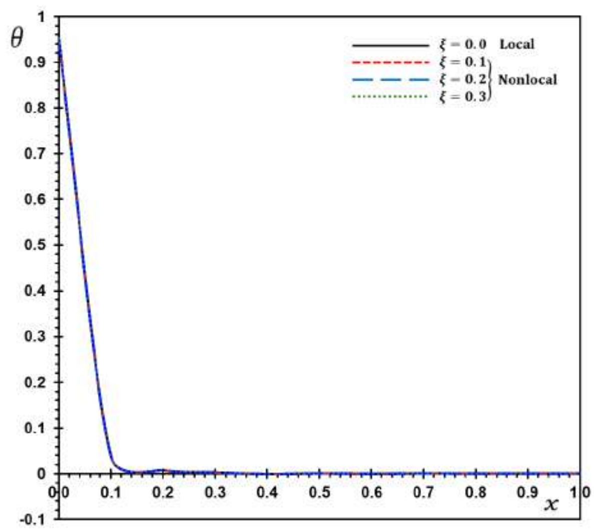

Fig. 3. The temperature θ with different nonlocal parameter ξ

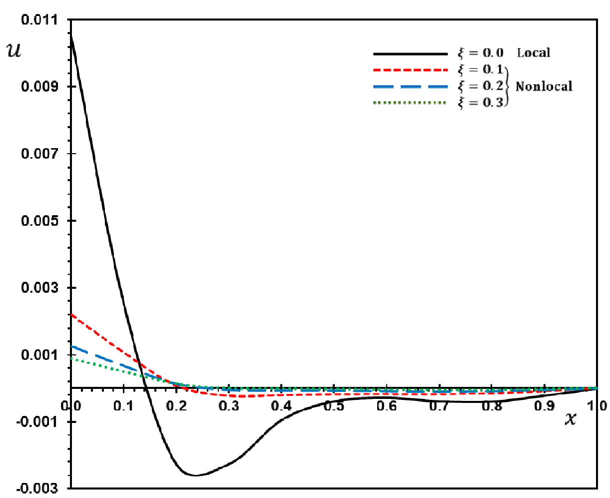

Fig. 4. The displacement u with different nonlocal parameter ξ

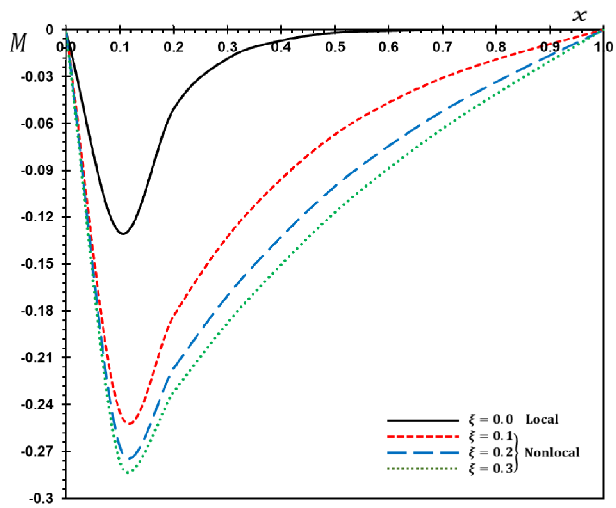

Fig. 5. The flexure moment M with different nonlocal parameter ξ

8.1 The effect of the nonlocal parameter ξ

The importance of the nonlocal parameter ξ is discussed in this subsection. The response of the lateral vibration, temperature, displacement, bending moment fields with scale coefficient is depicted Figs. (2–5). We can notice that when the nonlocal parameter ξ; vanishing $\xi = 0.0$ indicates the old situation (local model of elasticity) while other values indicate the nonlocal theories of elasticity and thermoelasticity. One may also notice from Figs. (2–5) that the parameter ξ has a significant effect on all the fields. This result is consistent with the results obtained by Zenkour and Abouelregal in [5, 6]. Also, it can be deduced that the thermal and elastic waves reach a steady-state depending on the nonlocal parameter values [39]. Furthermore, the obtained results are compared with those obtained by [40]. It can be observed that the results obtained well consistent with those provided by [40], for various non-local parameter values. The concluding remarks from the Figures can be shortened as follows:

- Figures. 2 and 5 show that the lateral vibration w increase when the value of ξ increases, while the values of the bending moment M decrease when the value of ξ increases.
- It can be seen from Fig. 3 that the temperature θ does not change by changing the nonlocal parameter ξ. Our findings are in strong accordance with the results of Abouelregal and Zenkour's study [38, 39].

From Fig. 5, the values of the displacement u decrease with increasing ξ in the range $0.0 \le x \le 0.4$, thereafter the variation became very small in the range $0.4 \le x \le 1.0$. This shows the distinction between the local generalized thermoelasticity and the nonlocal generalized thermoelasticity models.

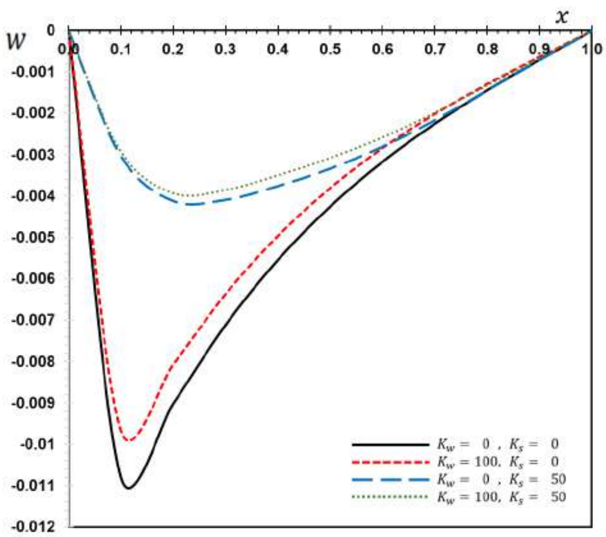

Fig. 6. The lateral vibration w with different foundation (K_w, K_s)

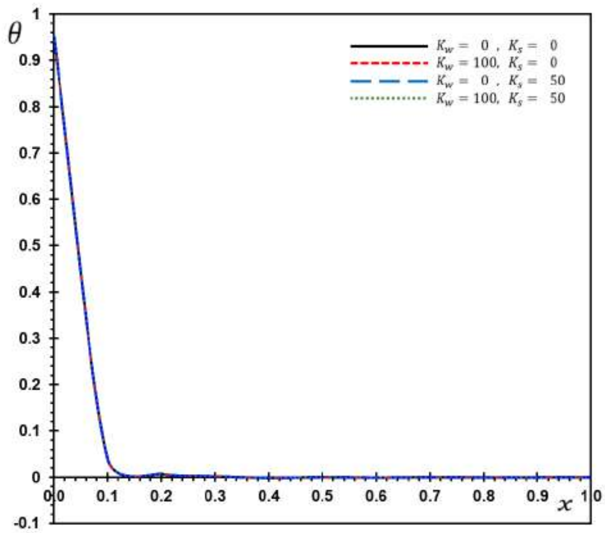

Fig. 7. The temperature θ with different foundation (K_w, K_s)

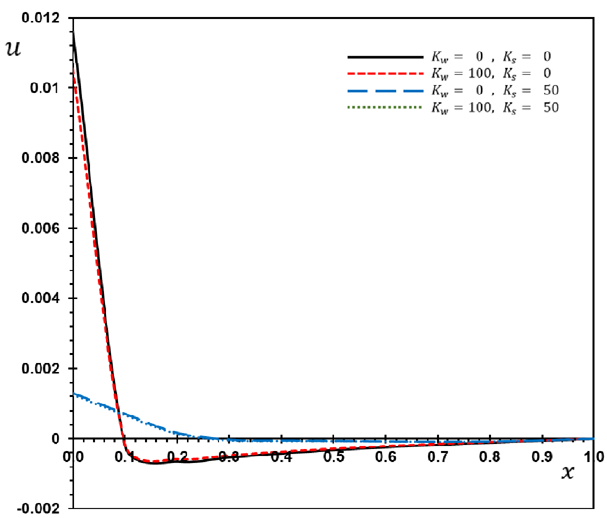

Fig. 8. The displacement u with different foundation (K_w, K_s)

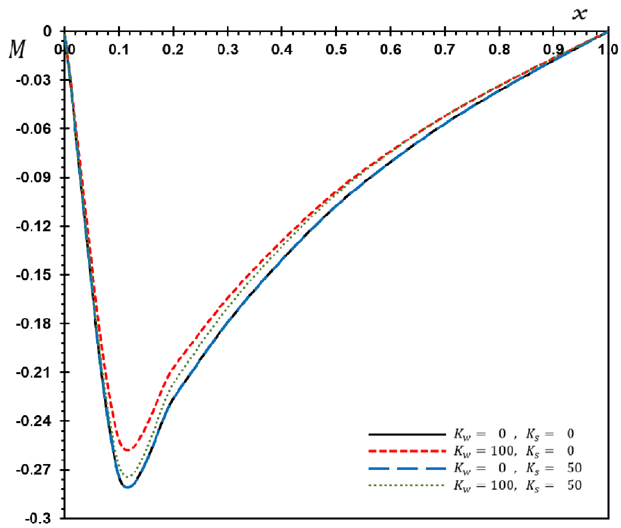

Fig. 9. The flexure moment M with different foundation (K_w, K_s)

8.2 The effect of the foundation parameters (K_w, K_s)

This case illustrates how the field quantities vary with various values of the foundation parameters (K_w, K_s) when $\xi = 0.2$. The numerical results are obtained and presented graphically in Figs. (6-9). Yokoyama [41] considered the free transverse vibration of the Euler–Bernoulli beam on a Winkler–Pasternak foundation. Togun et al. [42] investigated the nonlinear vibration of a nanobeam resting on a Winkler–Pasternak foundation based on Euler–Bernoulli beam theory. A comparative study was conducted to validate the present study. It can be seen from the figures that there is good harmony among the four results. This occurs due to an improvement in nanobeam rigidity when it is rested on elastic foundation. We can see the significant effect of the foundation parameters K_w and K_s on the lateral vibration w, displacement u and flexure moment M. Also, we can conclude that:

- In Fig. 6, the lateral vibration w increases when one or both of the foundations K_w and K_s are increased (the effect of K_s is greater than K_w) while in Fig. 9, the flexure moment M increases when the foundation K_w is increased (the effect of K_w is greater than K_s).
- The temperature θ does not vary with varying the foundations K_w or K_s (as shown in Fig. 7).

In Fig. 8, the values of the displacement u start decreasing with the foundations K_w and K_s in the range $0.0 \le x \le 0.4$, thereafter the variation became very small in the range $0.4 \le x \le 1.0$.

8.3 The effect of the ramp time parameter t_0

This case is studying how the non-dimensional lateral vibration w, temperature θ, displacement u and flexure moment M vary with ramp time parameter t_0 (see Figs. 10 -13). We have compared the findings of Abouelregal and Mohammed [44] and Abouelregal and Marin [45] to validate the current analytical results on the thermoelastic response of a dynamic nonlocal nanobeam. This result is consistent with the results obtained by [43-45].

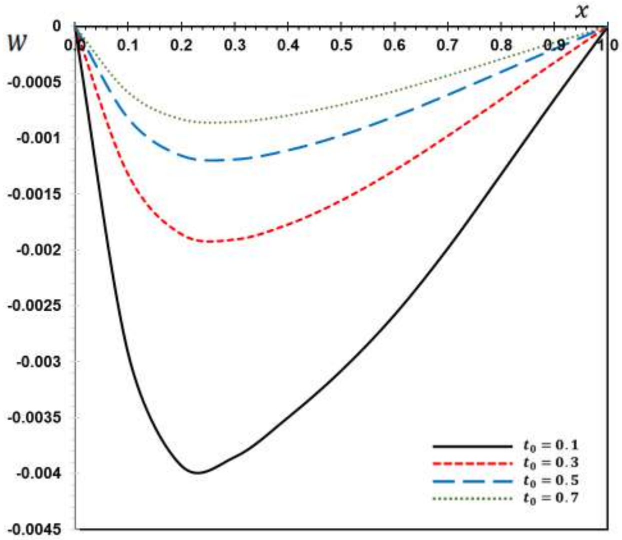

Fig. 10. The lateral vibration w with different ramp time parameter t_0

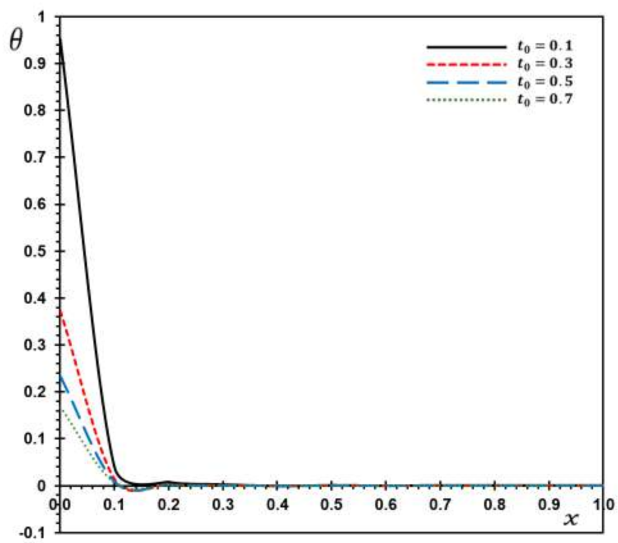

Fig. 11 The temperature θ with different ramp time parameter t_0

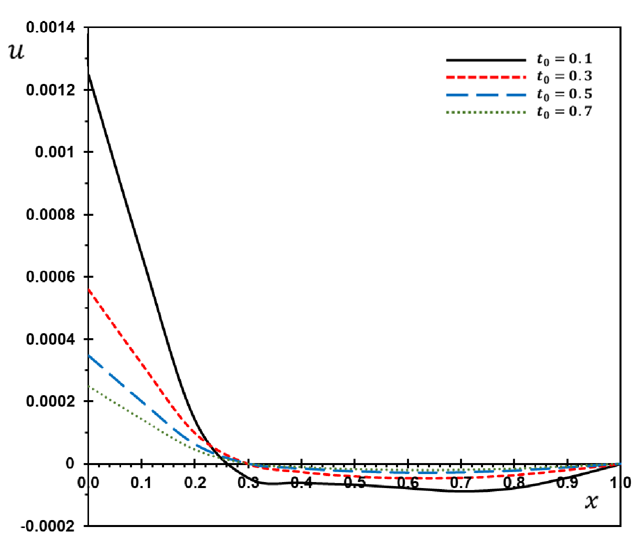

Fig. 12. The displacement u with different ramp time parameter t_0

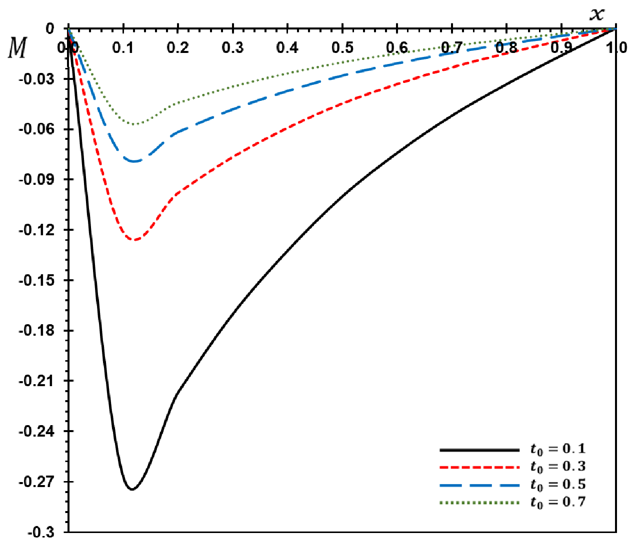

Fig. 13. The flexure moment M with different ramp time parameter t_0

The conclusions from Figs. (10 -13) are as follows:

- Figs. 10 and 13 show that the lateral vibration w and flexure moment M increase with increasing ramp time parameter t_0 .
- From Fig.11, The temperature θ decrease with increasing ramp time parameter t_0 in the range $0.0 \le x \le 0.2$, thereafter the temperature θ does not change in the range $0.2 \le x \le 1.0$.

In Fig. 12, the displacement u start to decrease with increasing ramp time parameter t_0 in the range $0.0 \le x \le 0.2$, thereafter the variation became very small in the range $0.2 \le x \le 1.0$.

8.4 The effect of the instantaneous time t

The fourth case is studying distribution of the non-dimensional lateral vibration w, temperature θ, displacement u and flexure moment M along the axial x and instantaneous time t of the moving nanobeam (see Figs. 14 -17). In these figures, we find that all the studied fields vary with the instantaneous time t and axial x. From Figs. (14 -17), the distributions all field quantities achieve their limiting values and satisfy the initial and boundary conditions.

9. Conclusion

In the current work, the governing equations of nonlocal nanobeams embedded in the two-parameter foundation are constructed based on the non-local Euler-Bernoulli beam and generalized thermoelasticity with phase lag theories. The elastic foundation is modelled as a two-parameter Pasternak foundation. Also, the thermoelastic vibration of the temperature, deflection, displacement and bending moment of nanobeam subjected to ramp-type heating is discussed and investigated. The effects of the nonlocal parameter, elastic coefficient of the foundation and the shear layer foundation stiffness parameters on all the field variables have been shown and presented graphically. If the shear layer foundation stiffness is neglected, Pasternak foundation becomes a Winkler foundation. It can be observed that as the nonlocal and the elastic foundation parameters increase, the behaviour of the field variables increase. Furthermore, it should be noted that for the higher values of the Pasternak and Winkler coefficients, the dimensionless deflection and critical buckling load of nanobeam are reduced.

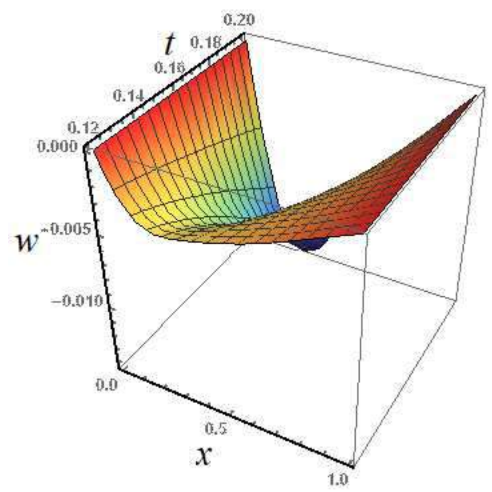

Fig. 14. Distribution of lateral vibration w along the axial x and time t

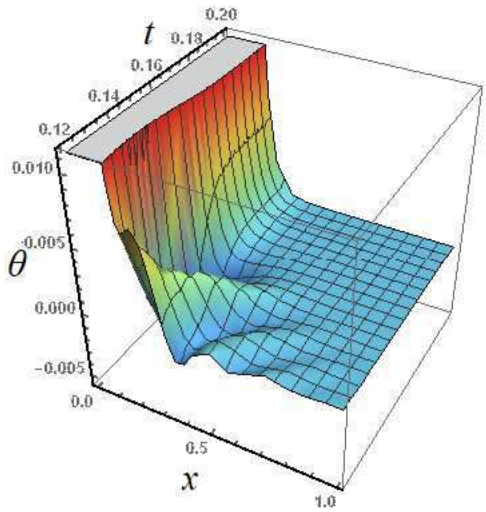

Fig. 15 Distribution of the temperature θ along the axial x and time t

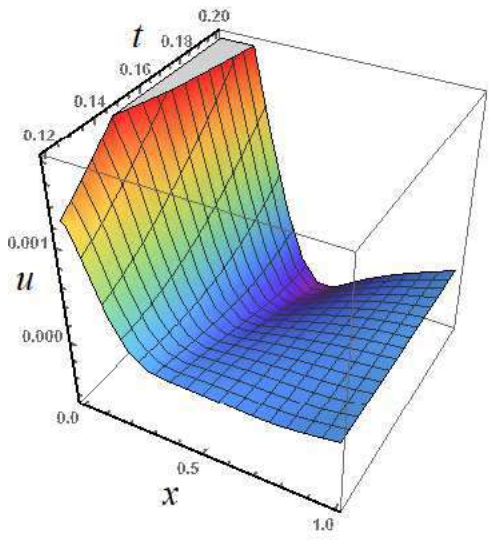

Fig. 16. Distribution of the displacement u along the axial x and time t

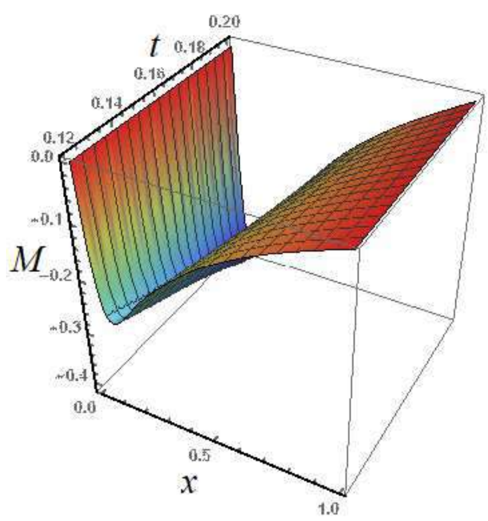

Fig. 17. Distribution of the flexure moment M along the axial x and time t

Author Contributions

All authors discussed the results, reviewed and approved the final version of the manuscript.

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Conflict of Interest

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Nomenclature

λ, μ	Lam´e's constants	K	Thermal conductivity
α_t	Thermal expansion coefficient	t	The time
$\gamma = E\alpha_t/(1-2\nu)$	Coupling parameter	q_i	Components of the heat flows vector
T_0	Environmental temperature	δ_{ij}	Kronecker's delta function
T	Absolute temperature	u_i	Displacement components
$\Theta = T - T_0$	Temperature increment	F_i	Body force components
C_E	Specific heat	Q	Heat source
τ_q	Phase lag of heat flux	τ_θ	Phase lag of gradient of temperature
σ_{ij}	Nonlocal stress tensor	h	Nanobeam thickness
e_{ij}	Strain tensor	ρ	Material density
L	Nanobeam length	b	Nanobeam width
$A = bh$	Cross–section area	oxyz	Cartesian coordinate
τ_{ij}	Local stress tensor	∇^2	Laplacian operator
ξ	Nonlocal parameter	E	Young's modulus
M_T	Thermal moment	e	Cubical dilatation
ν	Poisson's ratio	$I = bh^3/12$	Inertia moment

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ORCID iD

M.E. Nasr https://orcid.org/0000-0001-6003-4059
Ahmed E. Abouelregal https://orcid.org/0000-0003-3363-7924
A. Soleiman https://orcid.org/0000-0002-8544-457X
K.M. Khalil https://orcid.org/0000-0001-6732-1907

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